(1)



· General Information:

~ Procurement wants supply of RELIABILITY
Production UNIFORM CAPACITY UTILIZATION
Sales FAST & INDIVIDUAL FULFILLMENT OF CUSTOMER REQUIREMENTS
} High Inventory / large lot sizes

↳ Whereas Inventory Management wants Low Inventory levels, capacity reserves instead of stock...

↳ High Impact on corporate success => Stock in ROI...

· Inventory Functions:

BALANCING STOCK ↳ Bridging Time Lags
BUFFER STOCK ↳ Bridging Δ dimension Quantities in flow of goods
SAFETY STOCK ↳ Inventory to protect against S&D uncertainties
REFINEMENT STOCK ↳ Laytime adds value / part of production
SPECULATION STOCK ↳ Anticipation of price trends

↳ along the production process Inventories due to Δ in production rates

· Inventory Costs:

FIXED: · Setup / Maintain storage area ↳ do not depend on inventory level

ORDERING: · Managerial costs to prepare the purchase order (handling etc...)
↳ mostly fixed
=> Tendency: Large lot sizes, ordering cost p. unit ⊖

SETUP: · Prepare machine / process for manufacturing
=> Tendency: Large lot sizes

VARIABLE: · Costs from Storing / Holding Goods over a period of time
· ~ proportional to quantity of goods (CAPITAL COST)
=> Tendency for small lot sizes

SHORTAGE: · (Stockout) ↳ Demand not to be met due to low inventory

1. LOST SALES: · No product delivered => also effect on future sales

2. BACK ORDER: · Customer waiting for order => ⊕ handling, extra supply channel

· ABC-Analysis:

▸ Distinguishes the essential
▸ Focusing efforts
▸ Increasing efficiency of management

1.) Consumption Values
2.) Rank from highest to lowest, reorder
3.) Item % of Cons. Value then cumulate
4.) Item % of total item number
5.) ( % items | % value ) => Classify

A = High Value (70|20) B = Middle (20|30) C = Low (10|50)

(II)

# ⟶ ⊕ XYZ - Analysis ⇒ PREDICTION ACCURACY

$\{0 < CV \leq 10\}$ X - Items : Constant Consumption / Low Variation

$\{10 < CV \leq 25\}$ Y - Items : Notable fluctuation ⊕ trend like patterns

$\{25 < CV\}$ Z - Items : Items with complete irregular usage!

Predictability

⟶ Selection of Forecasting Method is crucial

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- Coefficient of variation:

⟶ Relative Standard Deviation (RSD)

$$\boxed{CV = \frac{SD}{\bar{x}} \cdot 100} \quad \left\{ SD = \sqrt{\frac{1}{n-1} \cdot \sum_{i=1}^{n} (x_i - \bar{x})^2} \quad \left\{ \bar{x} = \frac{1}{n} \cdot \sum_{i=1}^{n} x_i \right\} \right\}$$

⟶ Variations to a small mean evaluated bigger than to a larger mean!

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- Types of Procurement:

A) INDIVIDUAL
- ~ Procured only in response to concrete demand
- ~ High readiness for delivery from supplier
- ~ High risk of stockout

Graph

B) STOCK
- ~ Separation: Procurement / Production Schedule
- ~ good terms of delivery & buffer
- ~ Low Risk of stockout;

C) JIT
- ~ Procurement follows demand patterns
- ~ Sophisticated handling
- ~ Tapping into cost saving potential

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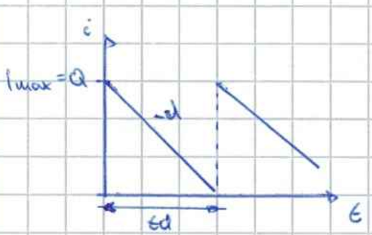
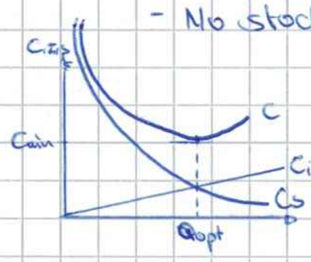
- Basic Model: Assumptions:
  - Disposition of 1 good
  - Demand is known, constant & independent
  - inventory from order arrives wholly at one point in time
  - Lead time is known and constant
  - No relevant capacity constraints
  - No stockouts & safety stocks




Inventory Usage over time




Cost

Optimal Q where total annual cost is a minimum.

- Quantity Discounts:

  - ~ Different prices per unit due to scale of order
  - ~ Unit price function of the order quantity

→ purchasing cost now to be considered in total cost function (decision - relevant!)

- Multiple Items & Storage:

  - now regarding multiple items
  - single item models applicable as long as there is no interaction between items
  - Items compete for space, Handling Time & financial resources

  ! Capacity constraints ∀ individual

Storage Types:

**Systematic**: ∈ Item own Space per Period

**Chaotic**: Space divided arbitrarily

Syst.-Chaotic: items → groups with own area → in areas chaotic

$$\sum_{i=1}^{M} S_{cap_i} \leq S_{cap}$$

$$a_i Q_i \leq S_{cap_i} \quad \forall i$$

$$\sum_{i=1}^{M} a_i Q_i \leq S_{cap}$$

$$\boxed{\text{Ø}} \quad \frac{1}{2} \cdot \sum_{i=1}^{M} a_i Q_i \leq S_{cap}$$ ↯ Storage demand not satisfied

$$\boxed{\Rightarrow} \quad \alpha \cdot \sum_{i=1}^{M} a_i Q_i \leq S_{cap}$$ ↯ storage area bottlenecks

⊖ Cost

$$0.5 \leq \alpha \leq 1$$

- Budget Constraint:

$$F_{cap} = \frac{GF_{cap}}{\text{mn}}$$

total budget

budget orders

↳ estim. mean # orders!

$$\sum_{i=1}^{M} p_i Q_i \leq F_{cap}$$

- Handling Constraint:

$$\sum_{i=1}^{M} h_i \cdot \underbrace{\frac{D_i}{Q_i}}_{\text{mn}} \leq H_{cap}$$

Demand of item

Model:

$$C(Q_i) = \sum_{i=1}^{M} \left( \frac{Q_i \cdot c_i}{2} + S_i \cdot \frac{D_i}{Q_i} \right) \to \min$$

$$\sum_{i=1}^{M} a_i Q_i \leq S_{cap}$$

$$\sum_{i=1}^{M} p_i Q_i \leq F_{cap}$$

$$\sum_{i=1}^{M} h_i \frac{D_i}{Q_i} \leq H_{cap}$$

$$Q_i \geq 0 \quad i \in \{1, \dots, M\}$$

BRUNNEN

II

⇝ Solving with LaGrange:

⇝ $3 \times \lambda + 3 \times SV_i$ ⇝ 8 derivations

⇝ yields: $Q_{i,20} = \sqrt{\frac{2 \cdot D_i (S + \lambda_3 h_i)}{c_i + 2\lambda_1 a_i + 2\lambda_2 p_i}}$

handling

capacity

price

ϟ Lagrange multiplicators unknown!!

$$= \sqrt{\frac{2D_i \cdot ek_{B_i}}{ek_{L_i}}}$$

---

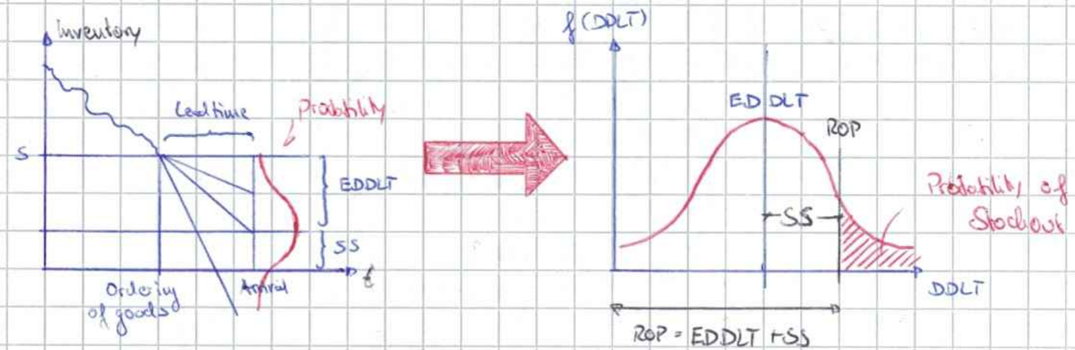
▹ From Basic to Advanced:

⇝ Now Considering uncertain demand
⇝ Risk of Stockouts

} Safety Stocks (SS)
⇒ When to order?

**ROP = EDDLT + SS**

Reorder Point = Expected demand during lead time + Safety Stock




ϟ Shortage costs exist but to what **EXTENT**?

⇒ Management sets ROP ("s") in order that α-service levels (no stockouts) are met.

⇒ the higher the SL the lower the stockout risk

ϟ **DDLT** often **NOT** following $N(0,1)$

⇒ $z = \frac{s-\mu}{\sigma}$ ⇔ $s = \mu(EDDLT) + \underbrace{z \cdot \sigma}_{=SS}$

ϟ Constant lead time and $N(0,1)$ demand **PER DAY**

(I) $\mu_{DDLT}$ = lead time in days $\cdot \mu_D$

(II) $\sigma_{DDLT} = \sqrt{LT \, \sigma_D^2}$

## ▹ Lot Size-reorder point Inventory Model:

A) $(t;q)$ Order policy: Order interval = const. Order quantity = const.

B) $(t,S)$ Order policy: Order interval = const. Order quantity = to target stock level "S"

**FOCUS** C) $(s,q)$ Order policy: ~~Order interval~~ ⇒ Control cycles; order if $i < s$ Order quantity = const

↳ Stockout possibly a thing

D) $(s,S)$ Order policy: Control cycles Order quantity = to "S"

E) $(T,s,q)$ Order policy: Control cycles = Order interval Order quantity = const.

F) $(T,s,S)$ Order policy: Control cycles = Order cycles Order quantity = to "S"

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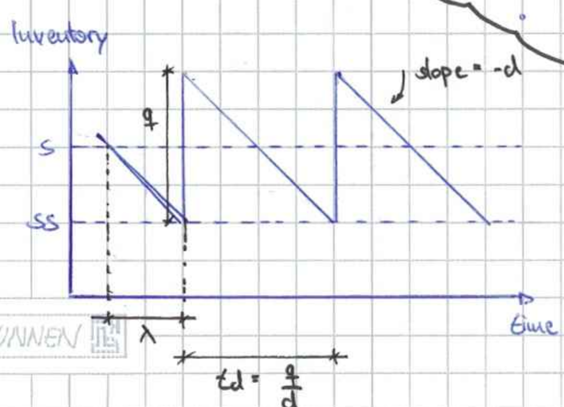
↳ **ASSUMPTIONS**:

- Order with "q" items placed when inventory level < "s"
- lead time "λ" deterministic & ≥ 0
- Demand rate "d" = const & > 0
  ⊕ $D = d \cdot T$
- Cumulated Demand during lead time "$D_\lambda$"
  → continuous random variable
  → probability density function $f(d_\lambda)$
  → cumulative distribution function $F(d_\lambda)$
  → μ denotes expected value

$d = \frac{\mu}{\lambda}$

- Shortages are allowed ⊕ delivered with delay
- unlimited storage capacity ⊕ continuous supervision
- $q$ - shortage $\overset{!}{>} s$ ⇒ no further orders




Mean Inventory level: $\frac{q}{2} + \underbrace{s - \mu}_{ss}$

Exp. cost of inventory: $E(C_I) = \left(\frac{q}{2} + s - \mu\right) cT$

↪ Shortage: $v(s) = \int_s^\infty (d_\lambda - s) \cdot f(d_\lambda)\, dd_\lambda$

→ because could occur in every order cycle & costs

↪ $E(K_F) = v(s) \cdot k_f \cdot \frac{D}{q}$

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↪ Optimal Order quantity:

$$E(C) = S \frac{D}{q} + \left(\frac{q}{2} + s - \mu\right) cT + v(s) \cdot k_f \cdot \frac{D}{q} \rightarrow \min$$

Ⓘ ↪ aus $\frac{\partial E(c)}{\partial q}$ $\Leftrightarrow$ $q^* = \sqrt{\frac{2D \cdot [S + v(s) \cdot k_f]}{c \cdot T}}$

Ⓘⓘ ↪ aus $\frac{\partial E(c)}{\partial s}$ $\Leftrightarrow$ $F(s^*) = 1 - \frac{c \cdot T \cdot q}{k_f \cdot D}$ ↪ $\overset{!}{>} 0$ & $\overset{!}{<} 1$

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↪ Iterative approach: Initiate: $q^*_0$ by Ⓘ with $k_f = 0$

Iteration: $s_i$ using Ⓘⓘ

$q^*_i$ using Ⓘ

a) opt $\alpha$: $F(s) = 1 - \frac{cTq}{q \cdot D}$

b) Exp. $v(s) = \sigma \cdot L(z)$

c) New $q^*$

STOP when $\Delta(q_{i+1}(s_{i+1}) - q_i(s_i)) < \epsilon$

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↪ Evaluation of $v(s)$: ... if $v(s) \sim N(0,1)$ => $v(s)$ via standard loss function

$$L(z) = \int_z^\infty (t - z)\, \phi(t)\, dt$$

$$\Rightarrow v(s) = \int_s^\infty (d_\lambda - s)\, f(d_\lambda)\, dd_y$$

↪ for a normal distributed DDLT: $v(s) = \sigma L(z) = \sigma \cdot L\left(\frac{s - M}{\sigma}\right)$

- $\alpha$ - Service-levels: no stocking out during lead time
- $\beta$ - Service-level: measures the proportion of demand not met during stock out.

→ $\alpha$: $F(s^*) = \alpha$ // $\alpha = 1 - \frac{c \cdot q}{k_f \cdot d}$

→ $\beta$: $\frac{v(s)}{q^*} = 1 - \beta$ $(D = q)$ // $\beta = w_s + \sum_{D_i > D_s} \frac{D_s}{D_i} \cdot w_i$

25.11.'19

Zsf. IM

• Service Level Determination for Style period orders:

• Meeting demand with one order (fashion, ... short lived products)

↯ Demand uncertain

## NEWSVENDOR-PROBLEM:

Total Exp. Profit: $EP(D_S) = \underbrace{(1 - W_{S-1}) \cdot (p - c_E) D_S}_{\text{Exp. profit } (D_i \geq D_S)} +$

$+ \sum_{i=1}^{S-1} \left[ \underbrace{(p - c_E)}_{\substack{\text{profit} \\ \text{per unit}}} D_i - \underbrace{(c_E - p_R)}_{\substack{\text{Loss per} \\ \text{unit } (D_i < D_S)}} \overbrace{(D_S - D_i)}^{\text{unsold units}} \right] w_i$

↳ Sorted into Payoff-Table ⇒ Exp. Values NEVER obtainable!

Expected long and short costs: $EC(D_S) = \sum_{i=1}^{S-1} (D_S - D_i) \underbrace{(c_E - p_R)}_{\substack{\text{loss per unit} \\ (D_i < D_S)}} w_i$ Cost of ord. too much LONG ($c_o$)

$+ \sum_{i=S+1}^{n} (D_i - D_S) \underbrace{(p - c_E)}_{\text{Profit per unit}} \cdot w_i$ Opport. cost SHORT ($c_{\bar{u}}$)

↳ Again in Payoff-Table

If $D_i \leq D_S$ $\quad \beta = 100\%$

$D_i > D_S$ $\quad \beta = W_S + \sum_{D_i \geq D_{S+1}} \frac{D_S}{D_i} \cdot w_i$

• Derivation of optimal α-Service-level:

• ⊕ 1 unit order ⇒ Sale occurs with $(1 - W_S)$ probability

↳ exp. profit $(1 - W_S) \cdot c_{\bar{u}}$

⇒ Remain unsold with $W_S$ probability

↳ exp. add. cost $(W_S) \cdot c_o$

↳ Optimal when equal: $W_S^* = \frac{c_{\bar{u}}}{c_o + c_{\bar{u}}}$

• Approximating Demand using N(0,1):

$\bar{D} = \frac{1}{n} \sum_{i=1}^{n} D_i$

$s^2 = \frac{1}{n-1} \sum_{i=1}^{n} (D_i - \bar{D})^2$

• Emp. probabilities = cont. record ↯
• multiple probabilities
• often to difficult to compute optimal inventory policies

$f(x) = \frac{1}{\sigma \cdot \sqrt{2\pi}} \cdot e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$

Crs optimal order quantity: $z = \frac{x-\mu}{\sigma}$

$Q^* = \sigma \cdot z + \mu \quad | F(z) = SL$

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▷ Contract classification: **BASFP PN** INVENTORY RISK ALLOCATION

**PUSH - MODELS**: Retailer takes risk — Wholesale price contract

**PULL - MODELS**: Supplier takes risk — Vendor managed inventory

**HYBRID - MODELS**: Shared risk:

↳ Indirect risk taken by supplier
- Buyback
- Revenue Sharing

↳ Direct risk taken by supplier
- Quantity flexibility
- APD

▶ Wholesale price contract:
~ Supplier produces/sells retailer's optimal order quantity w/o returning
~ retailer takes all the risk

▶ Vendor managed inventory (VMI):
~ Supplier's product shipped on consignment or Supplier holds inventory while replenishing retailer frequently
~ Supplier bears the risk

▶ Buyback Contract:
~ retailer takes risk by purchasing order quantity
~ supplier takes responsibility by buying back leftover stock
~ Supplier risk is indirect, i.e. goods are subject to damage

▶ Revenue Sharing Contract:
~ supplier sells at a discount
~ retailer shares the revenue

▶ Quantity flexibility:
~ forecast by the retailer
~ retailer can't purchase less than a cert. % under FC
~ supplier must guarantee to deliver a cert. % above FC
~ only one single price

▶ Advance-purchase discount:
~ prebook order (q) in advance of production ($w_1$ p.u.)
~ "at-once" orders when running out of stock (price higher)

02.12. Erg. IM

▷ Wholesale price contract:

- 2 stage supply chain (1 S & 1 R)
- retailer sells 1 season with stochastic demand
- price taker (p)
- orders in advance for w
- R produces after order with c as cost
- no replenishment

↪ retailer informed about w ⇒ based on w orders q (else q = 0)
↪ value of demand ("d") observed. (sales = min(q, d))

**OPTIMAL CONTRACT:**

→ Maximizes manuf. profit
→ R accepts every profit > opportunity cost (= 0)

---

▷ Model:

- 2 situations with stochastic demand ($D \lessgtr q$)
- Profit: $\pi_R(q)$

Retailer ↪ Critical Ratio: $F(q^*_R) = \frac{p-w}{p}$ $\left( \hat{=} \frac{c_{\bar{u}}}{c_o + c_{\bar{u}}} \mid c_o = \overset{=w}{c_E} - \overset{=0}{p_R} \mid c_{\bar{u}} = \overset{=p}{p} - \overset{=w}{c_E} \right)$

Retailer ↪ Manufacturers OF: $\pi_M(w) = (w-c)\, q^*_R(w) \rightarrow \text{Max}$

SC ↪ Critical Ratio: $F(q^*_{SC}) = \frac{p-c}{p}$ ($w = c$ → optimal & no profit)

Retailer ↪ Retailers Exp. Profit: $\pi_R(q) = (p-w)\,q - p \int_0^q (q-D)\, f(D)\, dD \rightarrow \text{Max}$

Retailer ↪ Opportunity Cost: $\pi_R(q) = \underbrace{(p-w)\cdot\mu}_{\text{exp. profit with no demand unc.}} - \underbrace{\left( w \int_0^q (q-D)\, f(D)\, dD + (p-w) \int_q^\infty (D-q)\, f(D)\, dD \right)}_{\text{Cost of demand uncertainty} \rightarrow \text{Opp. Cost}} \rightarrow \max$

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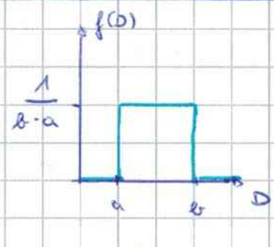
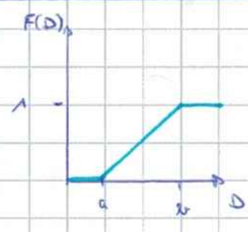
▷ Distribution functions:




$$f(D) = \begin{cases} \frac{1}{b-a} & ; a \le D \le b \\ 0 & ; D < a \text{ or } D > b \end{cases}$$

$$F(D) = \begin{cases} 0 & ; D < a \\ \frac{D-a}{b-a} & ; a \le D \le b \\ 1 & ; b < D \end{cases}$$

**MEAN:** $\frac{1}{2} \cdot (a+b)$

**VARIANCE:** $\frac{1}{12} (b-a)^2$

**COEFFICIENT OF VARIATION:** $\frac{1}{\sqrt{3}} \frac{b-a}{a+b}$

↳ retailer will choose the quantity, so that CDF = Crit. Ratio

↳ $\frac{q-a}{b-a} = \frac{p-w}{p} \Leftrightarrow q_R^* = b - \frac{w}{p}(b-a)$

↳ manufacturers exp. profit:

↳ $\overline{\pi}_M(w) = (w-c)\, q^*(w) \rightarrow \text{Max}$

↳ $w_M^* = \frac{bp}{2(b-a)} + \frac{c}{2}$ (OF is quadratic & concave in $[0,p]$)

$q_R^* = \frac{1}{2} \cdot \left(b - \frac{c \cdot (b-a)}{p}\right)$

$q_{SC}^* = b - \frac{c \cdot (b-a)}{p}$

! $0.5 \cdot q_{SC}^* = q_R^*$

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- Wholesale contract using a $N(0,1)$ ~distributed demand:

  - Assuming $f(D) = 0 \mid D < 0$
  - Negative tail = negligible : relative standard deviation $< 0{,}3$

  $[F(-z) = 1 - F(z)]$

↳ $\overline{\pi}_R(q) = (p-w) \cdot \mu - p\sigma\phi(z)$

---

- Buyback Contract:

↳ Contrasting the wholesale contract : supplier buys back leftovers for "r" ($\overline{\pi}_R$ + buyback profit)

↳ Critical ratio : $F(q_R^*) = \frac{p-w}{p-r}$ || $q_R^* = \frac{(p-w) \cdot b + (w-r) \cdot a}{p-r}$

$F(q_{SC}^{**}) = \frac{p-c}{p}$

↳ Manufacturer's OF: $\overline{\pi}_M(w,r) = (w-c) \cdot q_R^* - r\, q_R^*\, F(q_r^*) + r \cdot \int_0^{q_r^*} D\, f(D)\, dD \rightarrow \text{max!}$

Retailer's exp. profit: $\overline{\pi}_R(w,r) = (p-w) \cdot q + (p-r) \cdot \int_0^{q} D(f(D))\, dD - (p-r) \cdot q \cdot F(q)$

↳ Suppliers perspective: opt. when $r = w$

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- Supply Chain Coordination:

  - Assuming Retailer receives constant fraction $\lambda$ of SC Profit
  - SC coordinated if: $\lambda = \frac{(p-w)}{(p-c)}$ & $\lambda = \frac{p-r}{p}$
  - $\lambda$ = retailer's bargaining power

↳ $w_c^*(r) = \left(\frac{p-c}{p}\right) \cdot r + c$ | SC coordinated : respective profits dependent on bargaining power

# Zsf IM

## Basic Production Model (finite "p")

- Often several products, one machine
- lot-wise production (≙ quantity of a product, produced without interruption)

↯ Big lots = little setup cost BUT higher $C_I$

◎ Determining lot size that leads to minimal cost regarding counteracting effects

Either ▫ $P_1$ → finished goods $I$ → Demand

or ▫ $P_1$ → WIP inventory → $P_2$

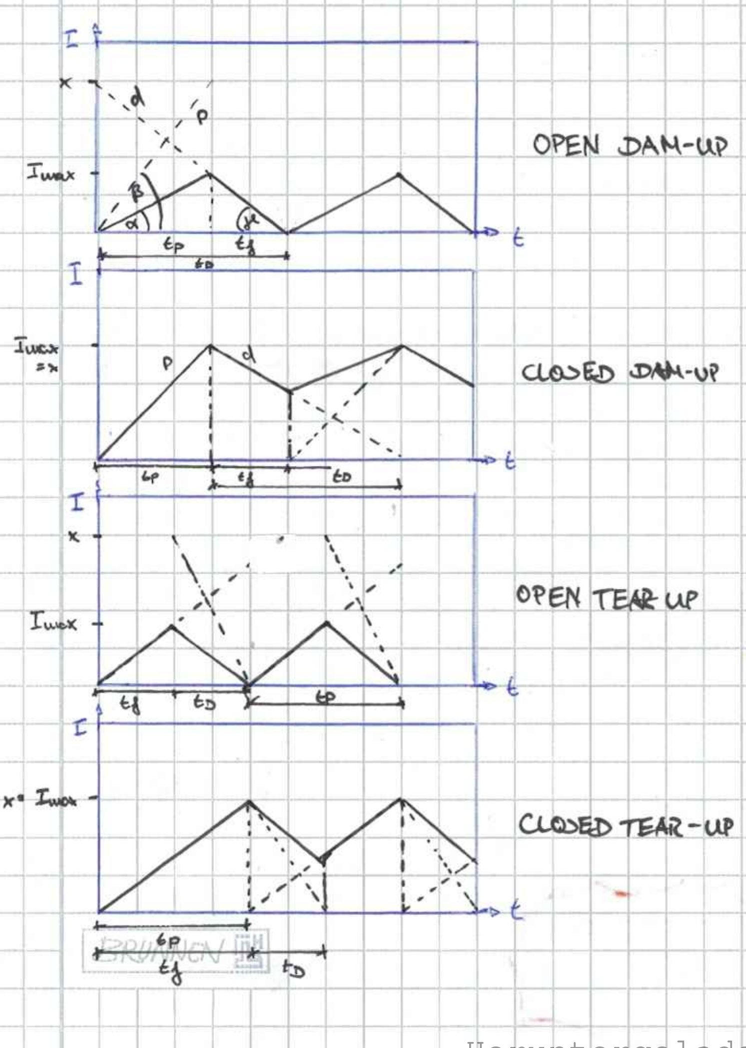
Either ▫ **Parallel Movement (Open)** Produced unit directly transferred

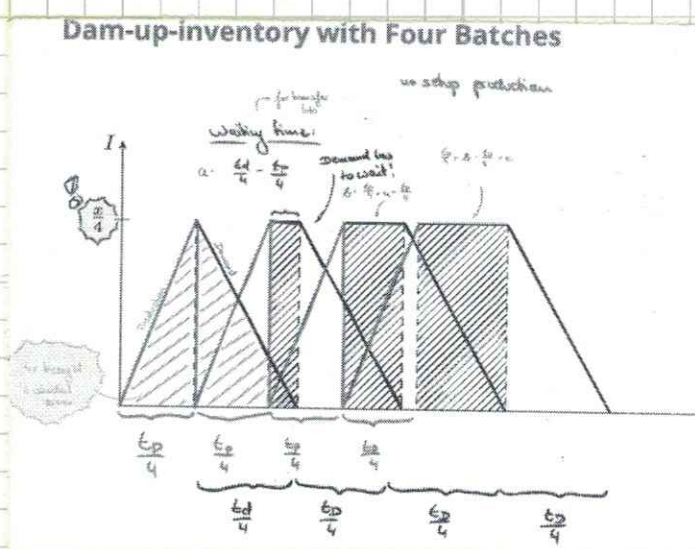
▫ Sequential -"- (Closed) Lot shipped to next stage when lot complete

## Production Process Classification:

| | OPEN-P. | CLOSED-P |
|---|---|---|
| DAM-UP-I ($p > d$) | Open Dam-up I | Closed Dam-UP I |
| TEAR-UP-I ($p < d$) | Open Tear-up I | Closed Tear-UP I |

$$\sqrt{\frac{2 \cdot S_{I/II}}{\left|\frac{1}{p} \mp \frac{1}{d}\right| \cdot c}}$$



▹ Inventory with equal sized batches:

⟹ Batches reduce Inventory Cost

⟹ with $\lim_{m \to \infty} C_I = 0$ ... Open Production

⟹ $C_{I,lot} = \frac{x}{m} \cdot \left(\frac{t_p}{m} + \frac{t_D}{m}\right) \cdot \frac{m}{2} + \frac{x}{m} \cdot \left(\frac{m-1}{2} \cdot (t_D - t_p)\right) \cdot c$

AREA OF TRIANGLES GREY AREA

$C_I = \frac{x}{2m} \cdot \left[ \left( \frac{1}{p} + \frac{1}{d} + (m-1) \cdot \left( \frac{1}{d} - \frac{1}{p} \right) \right] \cdot c \cdot D$

$C_S = k_R \cdot \frac{D}{x}$

$C_T = \underbrace{m \cdot k_T}_{\text{Trans Cost for 1 lot}} \cdot \frac{D}{x}$

$x_{opt}(\bar{m}) = \sqrt{\frac{2m \cdot (k_R + m \cdot k_T)}{\left[ \left( \frac{1}{p} + \frac{1}{d} \right) + (m-1) \cdot \left| \frac{1}{d} - \frac{1}{p} \right| \right] \cdot c}}$

= Tear-Up

⟹ Now batch number relevant for cost $C(x,m)$

▹ with $\alpha = \left(\frac{1}{p} + \frac{1}{d}\right)$ & $\beta = \left|\frac{1}{p} - \frac{1}{d}\right|$

I ⟹ $m_{opt} = \sqrt{\frac{(\alpha - \beta) \cdot k_R}{\beta \cdot k_T}}$ • NOT AN INTEGER VALUE •

II ⟹ $x_{opt} = \sqrt{\frac{2k_R}{\beta c}}$

▹ Convexity & Minimum of Cost Function:

- ① Quasi-convex

② But if both directional derivatives of twovariable function = 0 at given point ⟹ local minimum. (vgl. I, II)

⟹ If $x = x_{opt}$ & $m = m_{opt}$ ⟹ local minimum

⟹ $m_{opt} = \sqrt{\frac{\delta}{\varepsilon}}$

$\delta = (\alpha + (m-1)) \cdot c$ $\gamma = k_R + k_T \cdot m$

$\varepsilon = \frac{\beta c k_T}{2}$ $C(m) = 2 \cdot D \cdot \sqrt{\frac{\gamma \cdot \delta}{\varepsilon m}}$

$q = \frac{x}{m}$

↳ Or again(!) I

## INTEGER VALUE FOR $M_{opt}$?

! only (A) for $x_{opt}$, as we don't choose $m_{opt}$!

· Cost for $m_{opt} \pm h$ for $\forall h$ must be ≥ than $m_{opt}$

⟹ $\boxed{m^i_{opt} = \sqrt{\frac{\delta}{\varepsilon} + 0{,}25}}$ ... oder auch I + 0,25 ...

... then $x_{opt}$, $q$, $\gamma$ ...

▷ Unequal sized batches:

↳ No waiting time, sizing of lots makes waiting obsolete

↳ All lot sizes are dependend on $q_1$

- Lot size: $x = q_1 \cdot \frac{\left(\frac{p}{d}\right)^3 - 1}{\frac{p}{d} - 1} = q_1 \cdot A(m)$
- Lot Inv. Cost: $C_{I,lot} = \frac{q_1^2}{2} \cdot \left(\frac{1}{p} + \frac{1}{d}\right) \cdot \frac{\left(\frac{p}{d}\right)^6 - 1}{\left(\frac{p}{d}\right)^2 - 1} \cdot c$

$$= \frac{q_1^2}{2} \left(\frac{1}{p} + \frac{1}{d}\right) \cdot D(m = 3) \cdot c$$

- Cost function: $C(q_1, m) = \frac{q_1}{2} \cdot \underbrace{\left(\frac{1}{p} + \frac{1}{d}\right) \cdot D(m) \cdot c}_{\alpha} \cdot \frac{D}{A(m)}$

$$+ \underbrace{(S_R + m \cdot S_T)}_{\beta} \cdot \frac{D}{q_1 \cdot A(m)} \rightarrow \min$$

$$\Rightarrow q_1^{opt}(m) = \sqrt{\frac{2\beta}{\alpha}}$$

- $m_{opt}$: $C(m) = 2\sqrt{\left(\frac{1}{p} + \frac{1}{d}\right) \cdot D(m) \cdot c \cdot (S_R + m \cdot S_T)} \cdot \frac{D}{A(m)}$

↳ Compare for integer m-values, choose lowest
· quasi convex function

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▷ Common Cycle approach:

↳ Either open or closed BUT > 1 product on one machine

$$C(x) = \sum_{i=1}^{I} \frac{x_i}{2} \left(\frac{1}{p_i} + \frac{1}{d_i}\right) c_i D_i + \sum_{i=1}^{I} h_{Ri} \frac{D_i}{x_i} \rightarrow \min!$$

ASSUMPTION: All products have same number of production runs ("n")

↳ $n = \frac{D_i}{x_i}$

$$C(x) = \sum_{i=1}^{I} \frac{D_i^2}{2n} \left(\frac{1}{p_i} + \frac{1}{d_i}\right) c_i + n \cdot \sum_{i=1}^{I} h_{RI} \rightarrow \min!$$ (n only decision variable)

$$n_{opt} = \sqrt{\frac{\sum_{i=1}^{I} D_i^2 \left(\frac{1}{p_i} + \frac{1}{d_i}\right) \cdot c_i}{2 \cdot \sum_{i=1}^{I} h_{Ri}}}$$

$\sum_{i=1}^{I} (t_{Ri} + t_{pi}) \leq t_d$

($t_{D_A} \hat{=} t_{D_B}$ if $n_A \hat{=} n_B$)

BRUNNEN

⇒ ① Compute $x_{opt}$ like Open-Dam-Up

② Check feasibility via GANTT or $\min\{t_{li}\} \ge \sum (t_{R,i} + t_{P,i})$

⇒ Can calculate $n_{restr} = \dfrac{T - \sum_{i=1}^{I} \frac{D_i}{P_i}}{\sum_{i=1}^{I} t_{R,i}}$

**• FASTER •**

$n_{opt}$ & $n_{restr}$ → choose smaller one

because only $n_{opt}$ → $cost_{min}$ if $n_{restr} > n_{opt}$ logic and if $n_{opt}$ infeasible, $n_{restr} < n_{opt}$ and closest to optimum!

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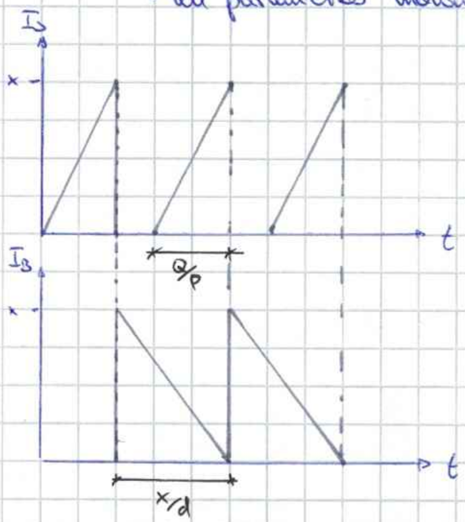
⇒ Joint Economic Lot Size Model (JELS):

• Coordination of the manufacturing policy of the supplier with buyer's ordering policy.

• Buyer periodically orders Q of item from supplier
• With receiving order supplier produces batch & ships
• FOCUS: Joint total relevant costs
• Optimal policy → cooperation
• $p > d$
• all parameters known & const.




• $C_S(x) = \underbrace{k_R \cdot \frac{D}{x}}_{\text{SETUP}} + \underbrace{\frac{x}{2} \cdot \frac{x}{p} \cdot c_S \cdot \frac{D}{x}}_{\text{INVENTORY}} \to \min!$

⇒ $x_{opt,S} = \sqrt{\frac{2k_R \cdot p}{c_S}}$ ; $C_{min,S} = 2D \cdot \sqrt{\frac{k_R c_S}{2p}}$

• $C_B(x) = \underbrace{S \cdot \frac{D}{x}}_{\text{ORDER}} + \underbrace{\frac{x}{2} \cdot \frac{x}{d} c_B \cdot \frac{D}{x}}_{\text{INVENTORY}} \to \min!$

⇒ $x_{opt,B} = \sqrt{\frac{2Sd}{c_B}}$ ; $C_{min,B} = 2D \cdot \sqrt{\frac{S c_B}{2d}}$

**RELATION:** $x_{opt,B} = x_{opt,S} \cdot \sqrt{\frac{\beta}{\alpha}}$ | $\beta = \frac{d c_S}{p c_B}$ ; $\alpha = \frac{k_R}{S}$

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**DOMINANCE:**

• If Supplier has market power: Dictating his own $x_{opt}$

• Otherwise supplier(s) have to accept $x_{opt,B}$

Higher Costs!

⇒ Joint planning $TC(x) = C_S(x) + C_B(x)$ ⇒ $x_{opt}^{j} = \sqrt{\dfrac{2 \cdot (k_R + S)}{\left(\frac{c_S}{p} + \frac{c_B}{d}\right)}}$

⇒ LYING: If pretense $k_R$ higher than actual ⇒ Condition S better!

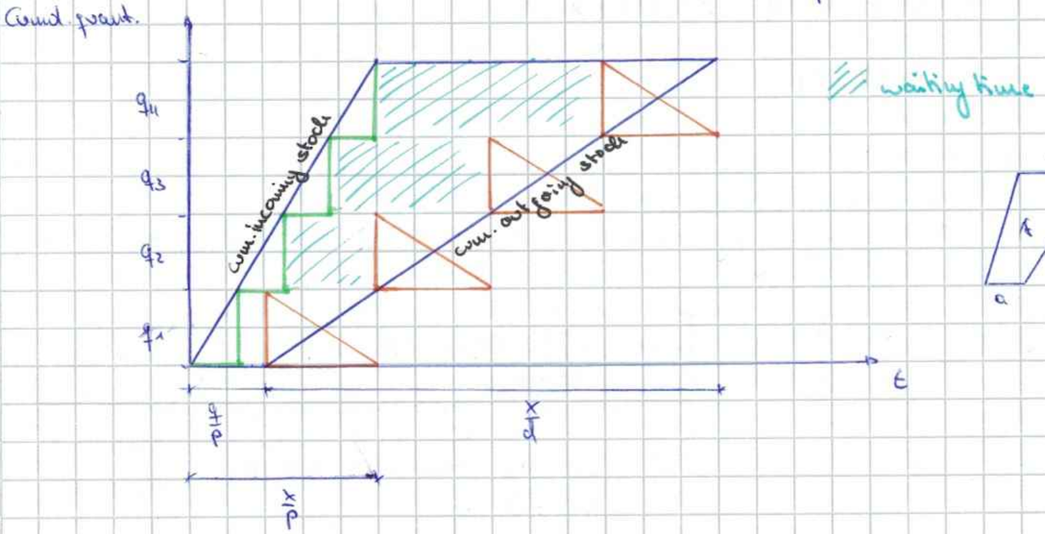
## ▹ Advanced JELS-Model (Joglekar):

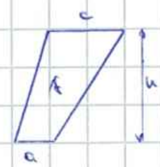
NEW

- New Coordination Mechanism (≠ lot for lot)
- lot size S (x) must be integer multiple of order size buyer (q) [$x = n \cdot q$]

  => S produces multiple, equally sized lots with one setup => successive transport

- Overall aim is collaborative optimization




$A = \frac{a+c}{2} h$

Area: $A = q \cdot [\frac{n}{2} \cdot (\frac{1}{d} - \frac{1}{p}) + \frac{1}{p}] \cdot D \cdot c_S = q \cdot \alpha \cdot D \cdot c_S$

Stock$_B$: $q \cdot \frac{1}{2d} \cdot D \cdot (c_B - c_S) = q \cdot \beta \cdot D \cdot (c_B - c_S)$

Cost: $C = q \cdot D [\alpha \cdot c_S + \beta \cdot (c_B - c_S)] + (\frac{u_B}{n} + n \cdot S) \cdot \frac{D}{q} \rightarrow$ min!

$= q \cdot D \cdot [\gamma] + \delta \cdot \frac{D}{q}$

$q_{opt} = \sqrt{\frac{\delta}{\gamma}}$ => $C(n) = 2D \cdot \sqrt{\gamma \cdot \delta}$

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## ▹ Deterministic Dynamic Inventory Lot-Size Model:

- T periods
- $D_t > 0$
- $Q_t$ is available immediately
- ⊗ Shortages
- ⊗ Capacity Constraints
- $I_1 = I_{T+1} = 0$
- $S \cdot q = c =$ const.

INVENTORY BALANCE: $I_{t+1} = I_t + Q_t - D_t$

NO SHORTAGES: $0 \leq Q_t \leq I_{t+1} + D_t \quad | I_t = 0$

COSTS IN PERIOD: $S + q \cdot Q_t + c\, I_{t+1} \quad | Q_t > 0$
$c\, I_{t+1} \quad | Q_t = 0$

CONDITION: $\sum_{t=1}^{T} Q_t = \sum_{t=1}^{T} D_t$

MODEL: OF: $\sum_{t=1}^{T} (S \cdot \underbrace{y_t(Q_t)}_{\text{Ordering}} + \underbrace{c I_{t+1}}_{\text{Inv. holding}}) \to \min!$

ST: $I_{t+1} = I_t + Q_t - D_t \quad \forall t$

$I_1 = I_{T+1} = 0 \quad \forall t$

$I_t \geq 0 \quad \forall t$

$0 \leq Q_t \leq I_{t+1} + D_t \quad \forall t$

WHERE: $y_t(Q_t) = \begin{cases} 1, & Q_t > 0 \\ 0, & Q_t = 0 \end{cases}$

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## PROBLEM SPECIFIC APPROACH

## WAGNER WITHIN

- Find Ordering policy$^{opt.}$ with corr. inventory levels
- Dynamic optimization problem (⇒ BELLMAN)
- Idea of placing an order in "b" with demands until period "l"

$$C_l = \min \left\{ \min_{1 \leq b < l} \underbrace{\left[ S + c \cdot \sum_{t=b+1}^{l} (t-b) \cdot D_t + C_{b-1} \right]}_{C_{I+O} \text{ of Demand ordered in } b} ; \underbrace{S + C_{l-1}}_{\text{Order of } b_t \text{ in } l} \right\}$$

... with $C_0 = 0$; $C_1 = S$

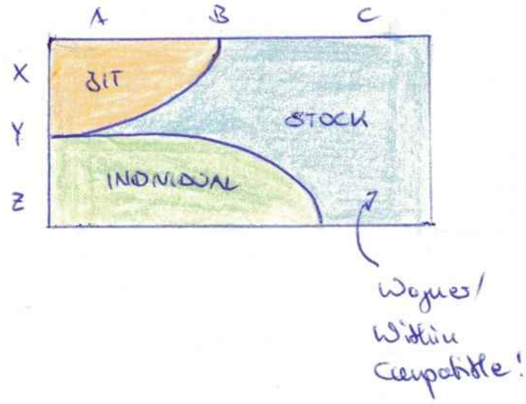
- Problem specific assumption:
  - Orders are placed only in periods where demand can't be met by the current inventory level.
  - Orders cover exactly demand of one or several periods

- THEOREM OF THE SHORTENED PLANNING HORIZON:

  If the demand D of period t can be covered by any order of $r_b$ units in period b optimally, the demands $D_b$, $D_{b+1}, \dots, D_{t-1}$ are covered optimally as well. As long as an order placed in period $l$ ($l < b$) is not more costefficient than an order in b, an order in l including demands beyond t can't provide less costs than an order in b.

ADVANTAGE: In a sequence of ordering decisions the previous ordering decisions are independent of the following ones as soon as the optimality of the directly following ordering period is confirmed.

## ABC & XYZ




## EQUAL SIZED BATCHES

- $\alpha = \left(\frac{1}{p} + \frac{1}{d}\right)$ & $\beta = \left(\frac{1}{p} - \frac{1}{d}\right)$

$$\Rightarrow m_{opt} = \sqrt{\frac{(\alpha - \beta) k_R}{\beta k_T} + \frac{1}{4}} \quad \hat{\uparrow} \downarrow \text{ for } i$$

$$\Rightarrow x_{opt} = \sqrt{\frac{2 k_R}{\beta c}}$$

## SUPPLY CHAIN COORDINATION

+ Buyback

$$\Rightarrow \lambda \cdot \pi_{SC} = \pi_R$$

$$\Leftrightarrow \lambda \cdot (p - c) = (p - w) \oplus \lambda \cdot p = (p - r)$$

$$\Leftrightarrow \frac{p - w}{p - c} = \frac{p - r}{p}$$

$$\Rightarrow w^r_c(r) = \left(\frac{p - c}{p}\right) r + c$$